

An undergraduate exercise in the First Law of Relativistic Thermodynamics

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Abstract

The isothermal compression of an ideal gas is analysed using a Relativistic Thermodynamics formalism based on the Principle of Inertia of Energy (Einstein's Equation) and the Asynchronous Formulation [G Cavalleri, and G Salgarelli, *Revision of the relativistic dynamics with variable rest mass and application to relativistic thermodynamics*, Nuovo Cimento **42**, 722-754 (1969)], which is similar to the formalism developed by Van Kampen [N G van Kampen, *Relativistic thermodynamics of moving systems*, Phys. Rev. **173**, 295-301 (1968)] and Hamity [V H Hamity, *Relativistic Thermodynamics* Phys. Rev. **187** 1745-1752 (1969)]. In this 4-vector Minkowski formalism mechanical and thermodynamical processes are described by the First Law of Thermodynamics expressed as $\Delta U^\mu = W^\mu + Q^\mu$, in a Lorentz covariant way. This exercise is considered useful for undergraduate physics students interested in foundations of physics, with the only prerequisites in first courses in thermodynamics and special relativity.

1 Introduction

The First Law of Thermodynamics $\Delta U = W + Q$, where ΔU is the increase in internal energy of the system, W is the work done by external forces and Q is the heat, is not, at first sight, covariant under a Galilean transformation [1]. Consider a vertical cylinder containing a gas and closed by a piston (Fig. 1). Cylinder and gas are at rest in the reference frame S_0 . A constant force $F_1 = -F$ is applied to the piston during time Δt . The displacement of this force is $\Delta y_1 = -\Delta y$ and the 'force-displacement product' (work) is $W = F_1 \Delta y_1 = F \Delta y$.

Let an observer be attached to the reference frame S_A – subindex A denotes magnitudes measured in the frame S_A –, which moves with vertical speed $-V$ respect to S_0 . In S_A , the internal energy is $\Delta U_A = \Delta U$, since the internal energy is frame independent. Energy transferred as heat – energy without

linear momentum associated to it –, is assumed to be invariant under Galilean transformations, $Q_A = Q$. Force is Galilean invariant, $F_{1A} = F_1$, and displacement transforms between frames S_0 and S_A as $\Delta y_{1A} = \Delta y_1 - V\Delta t$, where Δt is Galilean invariant. Thus, $W_A = F\Delta y - FV\Delta t$. Apparently, in frame S_A , $\Delta U_A \neq W_A + Q_A$ and the First Law of Thermodynamics does not seem to fulfil Galileo's Principle of Relativity.

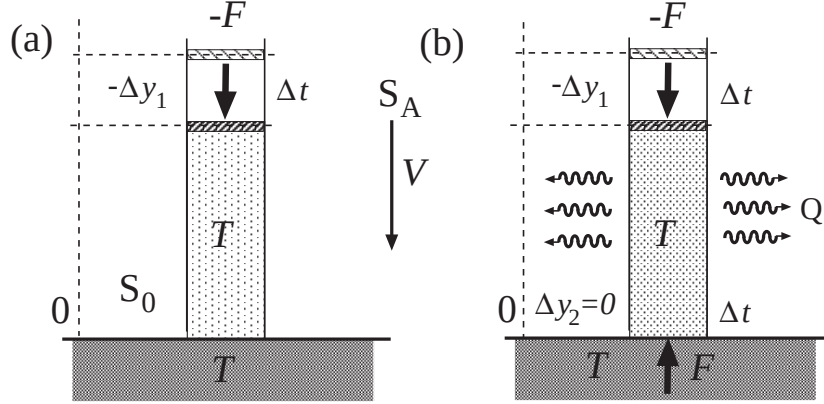


Figure 1: (a) A gas in a cylinder with diathermal walls is compressed by a vertical force $F_1 = -F$ on the piston. The displacement of the piston is $-\Delta y$ and the force acts during Δt . Heat Q is transferred to the surroundings through the walls. (b) Force F , on the floor, with no displacement, acts simultaneously during Δt . For an observer in the frame S_A the system as a whole moves with vertical velocity V .

In the previous description, a point related to Newton's Second Law has been omitted. In order to assure that the whole system remains at rest in frame S_0 , another vertical force, $F_2 = F$, without displacement, $\Delta y_2 = 0$, has to be applied to the bottom of the cylinder. The result is zero total force $\sum_j \mathbf{F}_j = 0$ and zero impulse, $\mathbf{I} = \left(\sum_j \mathbf{F}_j\right) \Delta t = 0$, on the system. Here Δt is the interval during which the vertical forces F_1 and F_2 are applied to the gas. In S_A , the force F_2 is displaced by $\Delta y_{2A} = -V\Delta t$ and the work related to its displacement is $W_{2A} = FV\Delta t$. Thus $W_A = W_{1A} + W_{2A} = \sum_j F_j \Delta y_{jA} = F\Delta y = W$. Then, $\Delta U_A = W_A + Q_A$ and the First Law is covariant under Galilean transformations.

In general, to solve this kind of exercise with forces applied on a composite, extended and deformable system, we have to use: (i) Newton's Second Law, $\sum_j \mathbf{F}_j = m \mathbf{a}_{\text{cm}}$, where m is the (constant) mass of the system and $\mathbf{a}_{\text{cm}}$ is the acceleration of the centre of mass, or, in integrated form (for constant forces),

the Centre-of-Mass Equation or Pseudowork Equation [2]:

$$\Delta K_{\text{cm}} = \left(\sum_j \mathbf{F}_j \right) \cdot \Delta \mathbf{r}_{\text{cm}},$$

where $K_{\text{cm}} = mv_{\text{cm}}^2/2$ is the kinetic energy of the center of mass, and $\Delta \mathbf{r}_{\text{cm}}$ is the displacement of the center of mass of the system, and (ii) the First Law of Thermodynamics [3]:

$$\Delta K_{\text{cm}} + \Delta U = W + Q,$$

with $\Delta E = \Delta K_{\text{cm}} + \Delta U$, where $E = K_{\text{cm}} + U$ is the total energy of the system.

For an observer in S_A , moving with velocity $\mathbf{V}$ with respect to S_0 ,

$$\Delta K_{\text{cmA}} = \left(\sum_j \mathbf{F}_{jA} \right) \cdot \Delta \mathbf{r}_{\text{cmA}}, \quad (1)$$

$$\Delta K_{\text{cmA}} + \Delta U_A = W_A + Q_A. \quad (2)$$

Under Galilean transformations – time Δt , forces $\mathbf{F}_j$, internal energy variations ΔU , and heat Q are all Galilean invariants –:

$$\begin{aligned} \Delta \mathbf{r}_{\text{cmA}} &= \Delta \mathbf{r}_{\text{cm}} - \mathbf{V} \Delta t, \\ \Delta K_{\text{cmA}} &= \Delta K_{\text{cm}} - \mathbf{V} \cdot \Delta \mathbf{p}_{\text{cm}}; \quad \Delta \mathbf{p}_{\text{cm}} = \Delta m \mathbf{v}_{\text{cm}}, \\ W_A &= W - \mathbf{V} \cdot \mathbf{I}; \quad \mathbf{I} = \sum_j \mathbf{F}_j \Delta t, \\ \mathbf{I} &= \Delta \mathbf{p}_{\text{cm}}; \quad \mathbf{I}_A = \Delta \mathbf{p}_{\text{cmA}} \end{aligned}$$

the set of Eqs. (1)-(2) is covariant under Galilean transformations. From these equations one obtains a new Galilean invariant [4]:

$$G = \Delta K_{\text{cm}} - W = \Delta K_{\text{cmA}} - W_A = -(\Delta U - Q).$$

1.1 Relativity and Thermodynamics

Let us now deal with a covariant description of this kind of problem in the formalism of Special Relativity [6], *i. e.*, under Lorentz transformations [1]. According to Rindler ([7] Sec. 1.10): ‘the unity of physics demands that all laws of physics must satisfy the Principle of Relativity and must be covariant under Lorentz transformations’.

From 1965 to 1970 Relativistic Thermodynamics aroused a remarkable interest among physicists. Yuen [5] wrote a good review showing the diversity of viewpoints on this issue in the early 70s.

The most complicated feature of Relativistic Thermodynamics is the necessity to harmonise the local nature of the equations of Special Relativity – which apply to point or elementary particles – and the non-local – extended

– nature of extensive thermodynamics quantities. The non-local character of deformable bodies – the perfect rigid body in Special Relativity does not exist –, is obvious when forces are applied to different points on a composite system.

The only way to warranty that physical laws, which are valid for elementary particles, can also be applied to extended systems – and that these laws remains Lorentz covariant – is to assure that the system behaves like a ‘single particle’, *i. e.*, that the system behaves as a whole [8]. The main goal of Relativistic Thermodynamics is to reach a unified description on point and extended-body dynamics.

In the Special Theory of Relativity, Lorentz transformations which describe how to relate observations of the same process in different frames are essential. [9]. In the context of Minkowski’s formalism of Special Relativity a magnitude with physical meaning should behave as a 4-vector under Lorentz transformation [10], *i. e.*, like the 4-vector interval $\Delta x^\mu = \{\Delta x, \Delta y, \Delta z, c\Delta t\}$ (Sec. 2).

In this work, the problem of the isothermal (non-quasistatic) compression of an ideal gas in the frame S_0 , in which the system is at rest, and in the frame S_A , in standard configuration to S_0 (Fig. 2), is solved by using 4-vectors – related by Lorentz transformations– being the First Law of Thermodynamics expressed in a Lorentz covariant form.

In Sec. 2 four-vector Minkowski’s algebra is briefly reviewed. In Sec. 3 the Principle of the Inertia of Energy is introduced. The energy function U and 4-vector U^μ are obtained for a mol of D_2 molecules enclosed in a cylinder. In Sec. 4 the original Asynchronous Formulation is reviewed. Work W and 4-vector W^μ are obtained for a set of forces applied on a deformable body. In Sec. 5 heat is modelled by a gas of thermal photons in a cavity. Heat Q and 4-vector Q^μ are obtained for the energy exchanged by a body. In Sec. 6 the First Law of Relativistic Thermodynamics is stated in Lorentz covariant form. In Sec. 7 we solve, as an exercise in Relativistic Thermodynamics, the isothermal compression of an ideal gas, in the frames S_0 and S_A . Finally, in Sec. 8 the conclusions are presented.

2 Four-vectors and Lorentz transformations

Consider two rigid reference frames S_0 and S_A , with identical units of length and time. The S_A origin moves with velocity $\mathbf{V} = (V, 0, 0)$ along the x -axis of S_0 , the x_A -axis coincides with the x -axis, while the y - and y_A -axes remain parallel, as do the z - and z_A -axes. All clocks are set to zero when the two origins coincide. The frames S_0 and S_A are said to be in standard configuration ([7] Sec. 1.3) (Fig. 2).

1. An event is described by $(\mathbf{x}, ct)$, $\mathbf{x} = (x, y, z)$, in frame S_0 and as

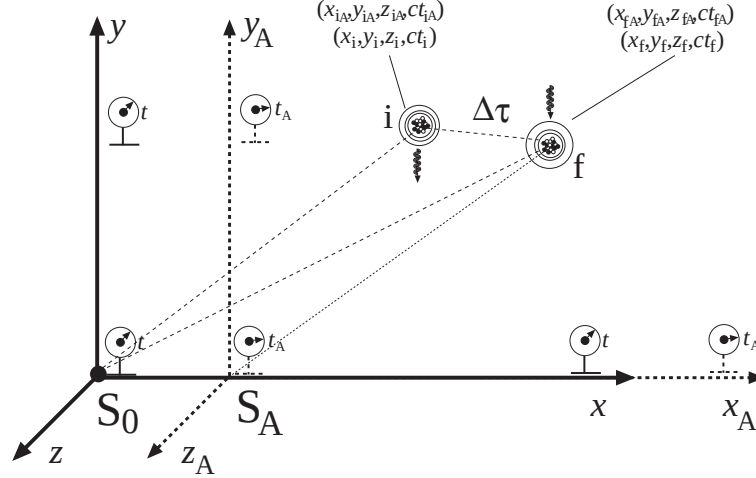


Figure 2: Frames S_0 and S_A in standard configuration. Axes y and z in both frames are parallel and frame S_A moves with velocity V along axis x of frame S_0 . At time $t = t_A = 0$ both origins coincide. Every frame has its own set of synchronised clocks. At event (i) an atom emits a photon. At event (f) the same atom absorbs a photon. The proper time $\Delta\tau$ between events (i) – $x_i^\mu = \{x_i, y_i, z_i, ct_i\}$ – and (f) – $x_f^\mu = \{x_f, y_f, z_f, ct_f\}$ – is the time measured by a clock moving with the atom.

$(\mathbf{x}_A, ct_A)$, $\mathbf{x}_A = (x_A, y_A, z_A)$, in frame S_A . This event is expressed as contravariant (greek index) 4-vectors [11]:

$$x^\mu = \begin{pmatrix} x \\ y \\ z \\ ct \end{pmatrix}; x_A^\mu = \begin{pmatrix} x_A \\ y_A \\ z_A \\ ct_A \end{pmatrix}$$

(A contravariant 4-vector may be written as a row 4-vector, keeping its contravariant index).

2. The Lorentz transformation for standard configuration velocity V is given by the mixed 4-tensor

$$\mathcal{L}_\nu^\mu(V) = \begin{pmatrix} \gamma(V) & 0 & 0 & -\beta(V)\gamma(V) \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\beta(V)\gamma(V) & 0 & 0 & \gamma(V) \end{pmatrix},$$

where $\beta(V) = V/c$ and $\gamma(V) = [1 - \beta^2(V)]^{-1/2}$ is the Lorentz factor.

Thus,

$$\begin{pmatrix} x_A \\ y_A \\ z_A \\ ct_A \end{pmatrix} = \mathcal{L}_\nu^\mu(V) \begin{pmatrix} x \\ y \\ z \\ ct \end{pmatrix}.$$

3. A relativistic invariant is a quantity with the same value in all inertial frames.
4. The raising and lowering of suffixes of 4-tensors is made by means of the metric tensor in Minkowski space ([12] Sec. 21) $g_{\nu\mu}$:

$$g_{\nu\mu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

The metric tensor will allow us to calculate inner products of 4-vectors.

5. According to Einstein's convention [13], wherever in any term of an expression a literal index occurs twice, this term should be summed up over all possible values of the index.
6. For a contravariant 4-vector A^μ , $A^\mu = \{A_x, A_y, A_z, A_t\}$, with spatial components the 3-vector $\mathbf{A} = (A_x, A_y, A_z)$ and temporal component A_t :
 - (a) A covariant (greek subindex, row) 4-vector $A_\mu = g_{\mu\nu}A^\nu$ is obtained by changing the sign of its spatial components, *i. e.*, $A_\mu = \{-A_x, -A_y, -A_z, A_t\}$.
 - (b) Physically measured magnitudes must be transformed between inertial frames using the Lorentz formula.
 - (c) Given a covariant 4-vector B_μ , with $B_\mu = \{B_x, B_y, B_z, B_t\}$, the inner product $B_\mu A^\mu$ – projection of A^μ along B^μ – is [14]:

$$B_\mu A^\mu = B_t A_t + B_x A_x + B_y A_y + B_z A_z.$$

The inner product is a relativistic invariant.

- (d) Its *norm* $\|A^\mu\|$ is defined as ([14] Sec. 41)

$$\|A^\mu\| = (A_\mu A^\mu)^{1/2} = \left[A_t^2 - (A_x^2 + A_y^2 + A_z^2) \right]^{1/2}.$$

The norm of a 4-vector is invariant, *i. e.*, $\|A^\mu\| = \|A_A^\mu\|$.

- (e) A linear combination of two 4-vectors is a 4-vector. If C^μ is a given 4-vector, $D^\mu = aA^\mu + cC^\mu$, where a and c are constants, is a 4-vector.

(f) Two 4-vectors A^μ and B^μ are said to be equal if, for all j

$$A_j = B_j \quad (j = 1, 2, 3, 4).$$

The equality of two 4-vectors is an invariant property. Consequently, a 4-vector equation is invariant [15].

7. The displacement Δx^μ between events, $x_i^\mu = \{x_i, y_i, z_i, ct_i\}$ and $x_f^\mu = \{x_f, y_f, z_f, ct_f\}$, defined by $\Delta x^\mu = x_f^\mu - x_i^\mu$, is a 4-vector. The 3-vector velocity is $\mathbf{v} = (v_x, v_y, v_z)$, where $v_x = \Delta x / \Delta t$, $\Delta x = x_f - x_i$, etc.
8. The proper time $\Delta\tau$, measured by a clock that moves with the particle (Fig. 2) ([7] p. 65), along a displacement Δx^μ is given by

$$c\Delta\tau = \left[(c\Delta t)^2 - (\Delta^2 x + \Delta^2 y + \Delta^2 z) \right]^{1/2},$$

or $c\Delta\tau = ||\Delta x^\mu||$. Thus,

$$(\Delta\tau)^2 = (\Delta t)^2 \left[1 - \frac{v^2}{c^2} \right]; \quad \frac{\Delta t}{\Delta\tau} = \gamma(v).$$

This equation expresses $\Delta\tau$ in terms of the time interval Δt measured in frame S_0 , which was chosen to describe the process.

9. The derivative of a 4-vector ΔA^μ with respect to its proper time is another 4-vector:

$$B^\mu = \frac{\Delta A^\mu}{\Delta\tau} = \frac{\Delta t}{\Delta\tau} \frac{\Delta A^\mu}{\Delta t} = \gamma(v) \frac{\Delta A^\mu}{\Delta t}.$$

The 4-vector velocity v^μ is defined by [16]:

$$v^\mu = \frac{\Delta x^\mu}{\Delta\tau} = \gamma(v) \{v_x, v_y, v_z, c\}.$$

3 Energy Function

The calculation of a 4-vector energy function U^μ [17] for a body demands the previous calculation of the energy function U [18] or total relativistic energy in frame S_0 , in which the system is at rest. The energy function U is obtained from Einstein's Equation $E_0 = mc^2$.

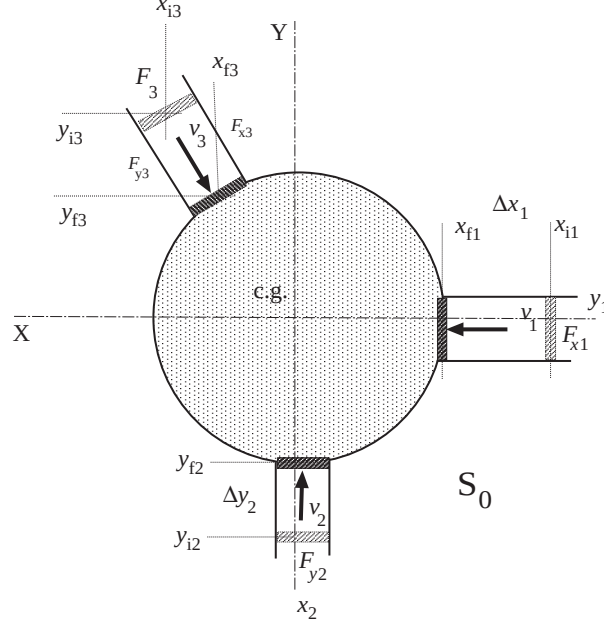


Figure 3: Forces F_k ($k = 1, 2, 3$) simultaneously applied on a body in frame S_0 with zero total impulse and zero torque. The system is at rest – its center of gravity (c. g.) remains at rest – and in thermal equilibrium.

3.1 Inertia of Energy

Consider a composite, extended, deformable body. Einstein's Equation $E_0 = mc^2$ [19] is expressed as the *Principle of the Inertia of Energy* [20]:

For an extended system in equilibrium, any kind of energy inside it, relativistically expressed in frame S_0 in which the system as a whole is at rest, contributes to the *energy function* U of the system. The *inertia* $\mathcal{M}$ of a composite system in equilibrium is:

$$\mathcal{M} = U c^{-2}. \quad (3)$$

This Principle relates the *inertia* $\mathcal{M}$ – the reluctance of the body to undergo a change in velocity [21] – with its *energy function* U – total energy content, including the mass of its elementary particle constituents, binding energies, electrostatic, chemical, internal kinetic energy, etc. [22] – through Einstein's Equation. All forms of energy have inertia [23].

In frame S_0 , in which the body is at rest and in equilibrium, the energy function U is a well-defined function and the inertia $\mathcal{M}$ of the body may be obtained. This inertia $\mathcal{M}$ enters in the definition of linear momentum p_A for the system in frame S_A , in which the body moves with velocity $-V$, as $p_A = -\gamma(V)\mathcal{M}V$. This relationship between inertia of energy and linear

momentum will become apparent when a Lorentz transformation is applied between different frames [17].

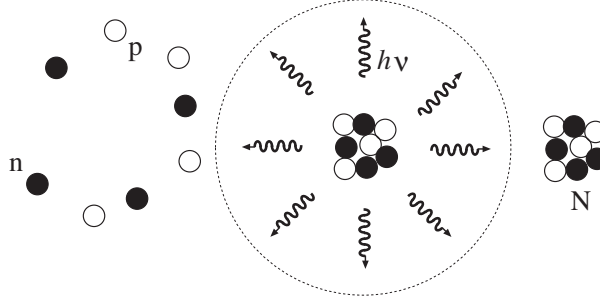


Figure 4: Formation of nucleus N from its components, protons, p, and neutrons, n, previously separated. The total energy of the nucleus U_N , and its inertia $\mathcal{M}_N = U_N c^{-2}$ is less than the sum of the energy of its components $\mathcal{U} = 2m_p c^2 + 2m_n c^2$. This decrement is equal to the energy released as emitted photons, $U_N - \mathcal{U} = -8h\nu$, when the nucleus is formed.

The energy function of a complex composite body is obtained through a recursive procedure.

1. Universal constants (c , h , k_B , N_A , etc.) are relativistic invariants. Number of atoms, photons, etc., inside a system are relativistic invariants [24].
2. For protons, neutrons or electrons its inertia equals its tabulated mass $-m_p$, m_n , m_e , respectively.
3. For a nucleus with $Z = n_p$ protons and $A - Z = n_n$ neutrons (Fig. 4), its inertia $\mathcal{M}_N$ is equal to the sum of the inertia of its elementary particles minus its binding energy $\tilde{U}_N$, the energy released when the nucleus is formed after its elementary particles [25] are initially at infinite separation, divided by c^2 [26]:

$$U_N = (n_p m_p + n_n m_n) c^2 - |\tilde{U}_N| ; \mathcal{M}_N = U_N c^{-2} .$$

4. For an atom A, ${}^A_Z\text{A}$, its inertia is equal to the sum of the inertia of its nucleus and electrons minus the energy released when the atom is formed from its nucleus and electrons, divided by c^2 [27]:

$$U_A = U_N + n_p m_e c^2 - |\tilde{U}_A| ; \mathcal{M}_A = U_A c^{-2} .$$

For instance, for the helium atom

$$\mathcal{M}_{\text{He}} = 2(m_p + m_n + m_e) - c^{-2} |\tilde{U}_{\text{N,He}} + \tilde{U}_{\text{A,He}}| .$$

5. The inertia $\mathcal{M}_M$ of a molecule is equal to the sum of the inertia of its atoms minus the energy released when bonds are formed $\tilde{U}_M$ [28] divided by c^2 . For instance, for a deuterium molecule (D_2) see Sec. 3.2.
6. The inertia of a system formed by a set of non-interacting components [29] for example a gas of D_2 molecules (Sec. 3.2), is equal to the sum of the total energy of their components, divided by c^2 .
7. For thermal radiation (ensemble of photons) filling a cavity [30] its energy contributes to the total inertia of the system. Thermal radiation emitted by a body will be described as radiation in a cavity (See Sec. 5.1). Although this radiant energy propagates at the speed of light, its inertia is non-zero [31].

Given the variety of systems studied [32], we prefer to use the term *inertia* [33] instead of the term mass [34] in order to avoid confusion [35].

Based on the Principle of Inertia of Energy, we assume that a composite system with inertia $\mathcal{M} = U c^{-2}$ behaves as an elementary particle of mass $\mathcal{M}$ (*Principle of Similitude* [1]), where U is the total energy measured in the zero-momentum frame S_0 ([20] p. 163).

For a macroscopic body this energy function is temperature dependent $U \equiv U(T)$ (see Sec. 3.2), as is its inertia $\mathcal{M}(T) = U(T) c^{-2}$, according to the Principle of Inertia of Energy. For a body with well-defined energy function $U \equiv U(T)$ (no volume dependence is included in U), $\mathcal{M}(T) = U(T) c^{-2}$, in frame S_A , in which the body moves with speed $-V$, its linear momentum p_A and total energy E_A are given by:

$$\begin{aligned} p_A &= -\gamma(V) \mathcal{M}(T) V, \\ E_A &= \gamma(V) U(T). \end{aligned}$$

This set of equations generalises for a composite body the set of equations $p_A = -\gamma(V) m V$ and $E_A = \gamma(V) m c^2$ which apply to an elementary particle mass m and speed $-V$. The total energy $E_A = K_A + U$ [the kinetic energy K of a composite body is not defined until later, in Eq. (5)] may be expressed as:

$$E_A^2 = [U(T)]^2 + c^2 p_A^2 = [\mathcal{M}(T)]^2 c^4 + c^2 p_A^2, \quad (4)$$

Eq. (4) is the thermodynamical generalisation of equation $E^2 = m^2 c^4 + c^2 p^2$ for an elementary particle with mass m . Thus, for a body:

$$\mathcal{M}(T) = \left[E^2 c^{-4} - p^2 c^{-2} \right]^{1/2},$$

is a relativistic invariant (in the absence of interactions with its surroundings) [36].

For an elementary particle its kinetic energy K is given by $K = [\gamma(V) - 1] mc^2$. For a composite body its kinetic energy K , defined as $K = E - U(T)$, is:

$$K = \frac{c^2 p^2}{E + U(T)} = [\gamma(V) - 1] U(T) = [\gamma(V) - 1] \mathcal{M}(T) c^2. \quad (5)$$

3.2 Energy Function 4-vector U^μ

As an application we obtain the energy function U and the 4-vector U^μ of a system formed by 1 mol, N_A molecules, of D_2 at temperature T . For a D_2 molecule, the energy function is:

$$u = 2(m_p + m_n + m_e) c^2 - \left(|2\tilde{U}_{N,D}| + |2\tilde{U}_{A,D}| + |\tilde{U}_{M,D_2}| \right).$$

where $\tilde{U}_{N,D}$, $\tilde{U}_{A,D}$ and $\tilde{U}_{M,D_2}$ are the ^2D nucleus strong binding energy, ^2D atom electrostatic binding energy and D_2 molecular binding energy, respectively. These (negative) binding energies are the energy released, in the form of photons, when D_2 molecules are formed from its elementary components [37].

We assume that in frame S_0 , every D_2 molecule moves with speed v (the kinetic equivalent for atoms in a container of the monochromatic approximation for thermal radiation in a cavity [38]) – but with different velocity components $\mathbf{v} = (v_x, v_y, 0)$, $v^2 = v_x^2 + v_y^2$. In this approximation, $v(T) = aT^{1/2}$. Constant a is obtained by imposing $k = [\gamma(v) - 1] u = 3k_B T/2$, where k is the kinetic energy of a D_2 molecule.

Linear momentum $\mathbf{p}_j$ and total energy e_j for the j -th molecule are:

$$\begin{aligned} \mathbf{p}_j &= \gamma(v)(uc^{-2})\mathbf{v}, \\ e_j &= \gamma(v)u. \end{aligned}$$

For the whole gas, in S_0 :

$$\begin{aligned} \mathbf{p} &= \sum_j \mathbf{p}_j = 0, \\ E &= \sum_j e_j = N_A \gamma(v)u. \end{aligned}$$

In frame S_0 the total energy E is the energy function $U(T)$ of the system.

In frame S_0 the four components of the 4-vector energy-momentum U^μ are given by ([17], p. 321):

$$U^\mu = \{0, 0, 0, U\}.$$

The Lorentz transformation $\mathcal{L}_\nu^\mu(V)$ from the zero-momentum frame S_0 to a frame S_A for the 4-vector $U_A^\mu = \{cp_A, 0, 0, E_A\}$ is $U_A^\mu = \mathcal{L}_\nu^\mu(V)U^\nu$, with:

$$U_A^\mu = \{-\beta(V)\gamma(V)U, 0, 0, \gamma(V)U\}.$$

Thus, the total linear momentum, the total energy and the kinetic energy of the body, in S_A are, respectively:

$$\begin{aligned} p_A &= -\gamma(V)\mathcal{M}V; \quad \mathcal{M} = U c^{-2} \\ E_A &= \gamma(V)U, \\ K_A &= E_A - U = [\gamma(V) - 1]U, \end{aligned}$$

The Principle of Inertia of Energy is implicit in the equation

$$|p_A| = \frac{E_A}{c^2}V,$$

i. e., every form of energy contributes to the linear momentum of the composite system.

The 4-vector U^μ norm is:

$$||U_A^\mu|| = \left[E_A^2 - c^2 (p_x^2 + p_y^2 + p_z^2) \right]^{1/2} = U.$$

The energy function U of a system may be obtained as the invariant norm of the 4-vector U^μ in any reference frame.

4 Work

For a force $\mathbf{F} = (F_x, F_y, F_z)$ acting on an elementary particle which moves with velocity $\mathbf{v} = (v_x, v_y, v_z)$ the 4-vector Minkowski force F^μ is defined as ([20] p. 189):

$$cF^\mu = \gamma(v) \{ cF_x, cF_y, cF_z, \mathbf{F} \cdot \mathbf{v} \}; \quad \mathbf{F} \cdot \mathbf{v} = F_x v_x + F_y v_y + F_z v_z.$$

On the other hand, a rigid body does not exist in Relativity ([37] pp. 88-89). Every extended system deforms under the action of a force. As a consequence, an applied force $\mathbf{F}_k$ on a body has an associated time interval Δt , a 3-vector displacement $\Delta \mathbf{r} = (\Delta x, \Delta y, \Delta z)$, and a 3-vector velocity $\mathbf{v} = (v_{xk}, v_{yk}, v_{zk})$ (Fig. 3).

The definition of the 4-vector Minkowski force may be generalised for a composite system on which different forces are applied at different points. This generalisation is achieved through the Asynchronous Formulation of Relativistic Thermodynamics.

4.1 Asynchronous Formulation

The problem of the relativistic transformation of physical magnitudes between two inertial observers was clarified, among others, by Rohrlich [39], Gamba [10] and Cavalleri and Salgarelli [40]. These authors introduced the concept of Asynchronous Formulation [41] to develop a coherent Relativistic Thermodynamics.

According to Cavalleri and Salgarelli, a privileged observer has to exist, in frame S_0 , who performs experiments on a body that remains at rest in S_0 . Every flux of energy through the surface of the system, be it work or heat, is exchanged with zero total impulse. The existence of frame S_0 assures the correspondence between the relativistic and the classical descriptions in the limit of low velocities.

Suppose that an observer in frame S_0 obtains a 4-vector A^μ , related to some physical quantity in a composite system. Measurements are performed in S_0 at different space points of the composite system but all of them during the same interval Δt . If an observer in frame S_A wants to determine the same quantity, he or she must perform measurements at different space points and at different times, then combine the results and obtain the Lorentz transformed value $A_A^\mu = \mathcal{L}_V^\mu(V)A^\nu$. In other words, an observer in S_A does not perform an experiment similar to that of an observer in S_0 , but just translates the experiment done in S_0 to his or her own system. For this observer, due to the relativity of simultaneity, forces applied simultaneously in S_0 will be non-simultaneous in S_A (asynchronous processes) [42].

If an observer in S_A performs measurements at the same interval of time Δt_A , he or she is not measuring magnitudes for the same experiment as the observer in S_0 . For instance, this occurs in the Lorentz contraction effect [43]. This result, denoted by $\tilde{A}^\mu$, differs from A_A^μ , and it is not related to A^μ by a Lorentz transformation [5]. According to Gamba, in the Asynchronous Formulation, observers in S_0 and S_A refer their own physical magnitudes, expressed as 4-vectors, to the *same* experiment (observed in frame S_0), and both descriptions of the experiment are related by *true* Lorentz transformations [39].

In Special Relativity the equilibrium condition for extended bodies is:

In the inertial frame S_0 a body is in equilibrium when the resultant of forces and torques, measured simultaneously, vanish. The translation of these conditions to conditions valid in another inertial frame S_A must be carried out in Minkowski space in terms of 4-vectors and Lorentz transformations [44].

4.2 Four-vector work W^μ

Consider in frame S_0 a set of k forces $\mathbf{F}_k = (F_{xk}, F_{yk}, F_{zk})$, applied at different points, by a work reservoir on an extended body (Fig. 5). These forces are applied during the same interval time Δt . Moving under force $\mathbf{F}_k$ the k -th piston has a 4-vector displacement $\Delta x_k^\mu = \{\Delta x_k, \Delta y_k, \Delta z_k, c\Delta t\}$, and a 4-vector velocity

$$v_k^\mu = \frac{\Delta x_k^\mu}{\Delta \tau_k} = \gamma(v_k) \{v_{xk}, v_{yk}, v_{zk}, c\} ,$$

with $\Delta t/\Delta \tau_k = \gamma(v_k)$, where $\Delta \tau_k$ is the proper time of the k -th piston [45].

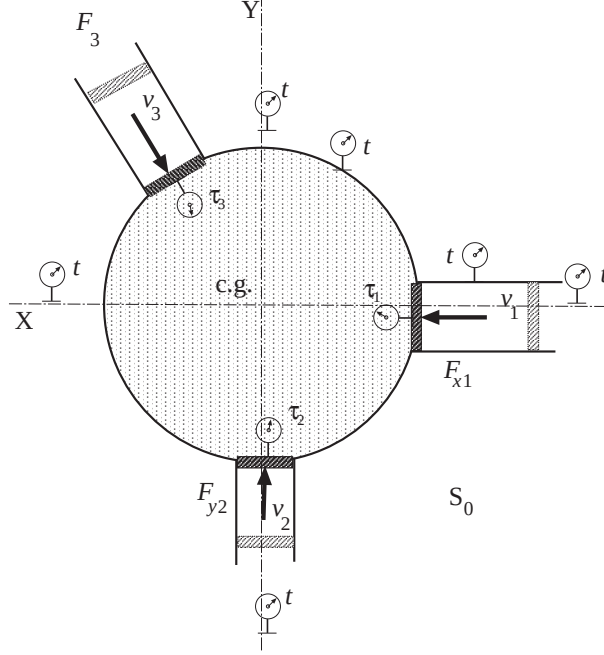


Figure 5: Set of forces F_k ($k = 1, 2, 3$) simultaneously applied on a system in frame S_0 (Fig. 3). The k -th piston moves $\Delta \mathbf{r}_k$ during time $t \equiv \Delta t$, as measured in S_0 , with speed v_k and proper time $\tau_k \equiv \Delta \tau_k$.

The impulse $\mathbf{I}_k$, $\mathbf{I}_k = \mathbf{F}_k \Delta t$ and the ‘force-displacement product’ [46] (work) W_k , $\mathbf{F}_k \cdot \Delta \mathbf{r}_k$, for the k -th piston are:

$$\begin{aligned} I_{xk} &= F_{xk} \Delta t, \\ I_{yk} &= F_{yk} \Delta t, \\ I_{zk} &= F_{zk} \Delta t, \\ W_k &= \mathbf{F}_k \cdot \Delta \mathbf{r}_k = F_{xk} \Delta x_k + F_{yk} \Delta y_k + F_{zk} \Delta z_k, \end{aligned}$$

Comparing with the force produced by an electric field $\mathbf{E} = (E_x, E_y, E_z)$ on a charged particle, with electric charge q , described by the 4-tensor $\mathcal{F}^{\nu\mu}$ ([47], p. 221; [12], Sec. 44):

$$\mathcal{F}^{\nu\mu} = q \begin{pmatrix} 0 & 0 & 0 & E_x \\ 0 & 0 & 0 & E_y \\ 0 & 0 & 0 & E_z \\ -E_x & -E_y & -E_z & 0 \end{pmatrix},$$

the 4-tensor $\mathcal{F}_k^{\nu\mu}$ –double contravariant– force field is given by ([12], Sec. 42):

$$\mathcal{F}_k^{\nu\mu} = \begin{pmatrix} 0 & 0 & 0 & F_{xk} \\ 0 & 0 & 0 & F_{yk} \\ 0 & 0 & 0 & F_{zk} \\ -F_{xk} & -F_{yk} & -F_{zk} & 0 \end{pmatrix}.$$

The mixed 4-tensor $\mathcal{F}_k^\mu{}_\nu$ is:

$$\mathcal{F}_k^\mu{}_\nu = g_{\nu\alpha} \mathcal{F}_k^{\alpha\mu} = \begin{pmatrix} 0 & 0 & 0 & F_{xk} \\ 0 & 0 & 0 & F_{yk} \\ 0 & 0 & 0 & F_{zk} \\ F_{xk} & F_{yk} & F_{zk} & 0 \end{pmatrix}.$$

For the k -th piston, it is possible to obtain two 4-vectors: (i) the 4-vector Minkowski force F_k^μ and (ii) the 4-vector work (impulse-work) W_k^μ .

(i) The 4-vector Minkowski force F_k^μ for the k -th force ([47], p. 167) is:

$$cF_k^\mu = \mathcal{F}_k^\mu{}_\nu v_k^\nu = \gamma(v_k) \{cF_{xk}, cF_{yk}, cF_{zk}, \mathbf{F}_k \cdot \mathbf{v}_k\},$$

$$\text{with } \mathbf{F}_k \cdot \mathbf{v}_k = F_{xk}v_{xk} + F_{yk}v_{yk} + F_{zk}v_{zk}.$$

(ii) The 4-vector work W_k^μ for k -th force is given by:

$$\begin{aligned} W_k^\mu &= \mathcal{F}_k^\mu{}_\nu \Delta x_k^\nu \\ &= \{cF_{xk}\Delta t, cF_{yk}\Delta t, cF_{zk}\Delta t, F_{xk}\Delta x_k + F_{yk}\Delta y_k + F_{zk}\Delta z_k\}, \\ &= \{cI_{xk}, cI_{yk}, cI_{zk}, W_k\}, \end{aligned}$$

a 4-vector with units of energy.

For the k -th piston, the 4-vector force F_k^μ is obtained by differentiating the 4-vector $W_k^\mu \equiv \delta W_k^\mu$ with respect to the proper time $\Delta\tau_k$:

$$\frac{\delta W_k^\mu}{\Delta\tau_k} = \frac{\Delta t}{\Delta\tau_k} \frac{\delta W^\mu}{\Delta t} = cF_k^\mu, \quad (6)$$

where $\Delta x_k/\Delta\tau_k = \gamma(v_k)v_{xk}$, etc. Eq. (6) assures that W_k^μ is, in fact, a 4-vector.

For the set of k forces simultaneously applied in frame S_0 during the interval Δt , on different pistons, one obtains a total 4-vector ‘force-displacement product’ (work) W^μ as the sum of the 4-vectors W_k^μ subject to the condition $\sum_k \mathbf{I}_k = 0$:

$$W^\mu = \{0, 0, 0, W\}; \quad W = \sum_k W_k.$$

In frame S_A , $W_A^\mu = \mathcal{L}_V^\mu(V)W^\nu = \{cI_{xA}, 0, 0, W_A\}$, with the impulse I_{xA} and the ‘force-displacement product’ W_A are:

$$\begin{aligned} I_{xA} &= -\gamma(V) (Wc^{-2}) V, \\ W_A &= \gamma(V) W, \end{aligned}$$

with

$$||W_A^\mu|| = [W_A^2 - c^2 \mathcal{I}_{xA}^2]^{1/2} = W.$$

5 Heat

In classical thermodynamics work is oriented energy transfer between a body and work reservoirs. Heat is considered to be random –i. e., ‘without linear momentum’– internal energy transfer between different bodies at different temperatures [48].

In Relativistic Thermodynamics, Van Kampen called Q_A^μ the *thermal energy-momentum transfer* 4-vector [49] and the scalar $Q = ||Q_A^\mu||$, the *heat supply* measured in S_0 , the rest frame of the system. With the 4-vector Q^μ given in S_0 as $Q^\mu = \{0, 0, 0, Q\}$, in frame S_A it becomes

$$Q_A^\mu = \mathcal{L}_\nu^\mu(V) Q^\nu = \left\{ -c\gamma(V)\frac{Q}{c^2}V, 0, 0, \gamma(V)Q \right\},$$

and a linear momentum p_A is associated with the heat Q , given by:

$$p_A = -\gamma(V)\frac{Q}{c^2}V.$$

The space components of 4-vector Q^μ – the associated linear momentum – are non-vanishing in inertial frames moving with respect to the privileged frame S_0 , in which these components are zero. The origin and the contribution to the First Law of this linear momentum of heat is not obvious. According to Rohrlich, the linear momentum of heat in frame S_A demands a physical interpretation [50]. Moreover, in the Special Theory of Relativity, any transfer of energy, being equivalent to a transfer of inertia, necessarily involves momentum [51]. This statement is valid for all forms of radiation and it must be valid for heat [52] whatever definition of it is used.

5.1 Thermal radiation in a cavity

In order to provide a relativistic interpretation of the linear momentum of heat and a relativistic characterisation of a thermal bath, we will describe thermal radiation as an ensemble of photons in a cavity [53].

For a photon with wavelength λ , angular frequency $\omega = 2\pi/\mathcal{T}$, period $\mathcal{T} = \lambda/c$, and frequency $\nu = c/\lambda$, that propagates in a direction given by the wave vector $\mathbf{k}$:

$$\mathbf{k} = \left(\frac{2\pi}{\lambda} \cos \theta, \frac{2\pi}{\lambda} \sin \theta, 0 \right),$$

(we assume that photons move in plane $x - y$) the 4-vector wave vector ω^μ is [54]:

$$\omega^\mu = \left\{ c\frac{2\pi}{\lambda} \cos \theta, c\frac{2\pi}{\lambda} \sin \theta, 0, \frac{2\pi}{\mathcal{T}} \right\}.$$

For a given ensemble of photons inside a cavity, with walls at temperature T , there is a frame S_0 in which these walls are at rest and the total linear momentum of the system of photons is zero. Such a cavity operates as a

thermal bath at temperature T . In the monochromatic approximation [38] photons in that cavity have frequency ν , with $\nu(T) = AT^{-1}$ (A is a constant related to Wien's Law).

In frame S_0 , the k -th photon, with frequency ν and moving in direction $\hat{\mathbf{k}} = (\cos \theta_k, \sin \theta_k, 0)$, has 4-vector $\omega_k^\mu = 2\pi \{\nu \cos \theta_k, \nu \sin \theta_k, 0, \nu\}$. Associated with it is an energy 4-vector, Q_k^μ , given by ($\hbar = (2\pi)^{-1}h$):

$$Q_k^\mu = \hbar \omega_k^\mu = \left\{ c \left[\frac{h\nu}{c} \right] \cos \theta_k, c \left[\frac{h\nu}{c} \right] \sin \theta_k, 0, h\nu \right\},$$

where $\mathbf{p}_p = (h\nu/c)\hat{\mathbf{k}}$ is the momentum of the photon and $E_p = h\nu$ its energy. The norm of Q_k^μ is $||Q_k^\mu|| = 0$. An individual photon has no inertia.

Ignoring fluctuations in the photon number, we assume that the number of photons inside the cavity is constant. In frame S_0 , the total linear momentum of N photons inside the cavity at temperature T , $\mathbf{p}_p = (p_{x_p}, p_{y_p}, 0)$, and the total energy E_p , are given by:

$$\begin{aligned} p_{x_p} &= \sum_k \frac{h\nu}{c} \cos \theta_k = 0, \\ p_{y_p} &= \sum_k \frac{h\nu}{c} \sin \theta_k = 0, \\ U_p(T) &= \sum_k h\nu = Nh\nu(T). \end{aligned}$$

For the ensemble of thermal photons inside the cavity, it is possible to construct a 4-vector $Q^\mu = \sum_k Q_k^\mu$ given by:

$$Q^\mu = \{0, 0, 0, U_p(T)\}; \quad U_p(T) = Nh\nu(T).$$

In frame S_A ,

$$Q_A^\mu = \mathcal{L}_\nu^\mu(V)Q^\nu = \{-\gamma(V)\beta(V)U_p(T), 0, 0, \gamma(V)U_p(T)\}.$$

The ensemble of thermal photons inside the cavity has an associated inertia $\mathcal{M} = U_p(T)c^{-2}$, [55], according to the Principle of Inertia of Energy.

5.2 Four-vector heat Q^μ

In the Asynchronous Formulation generalisation of heat, there should be a privileged frame, S_0 again, in which the system is at rest with respect to the thermal bath and in which thermal radiation (photons) are absorbed or emitted with zero total momentum. That demands that in S_0 , body and thermal reservoir are mutually at rest.

With the system and the thermal reservoir mutually at rest, the ensemble of photons emitted by a body – at a temperature T_1 higher than that of the thermal reservoir T – can be described as photons frequency $\nu(T_1)$ in a cavity,

with zero total linear momentum. In the same way, the set of absorbed photons –when the system is at a temperature T_2 lower than the thermal reservoir) can be described as an ensemble of photons with frequency $\nu(T)$ in a cavity with zero total linear momentum.

In frame S_0 , the 4-vector *thermal radiation* Q^μ when the system absorbs (+) or emits (-), N photons with frequency $\nu(T)$, is given by:

$$Q^\mu = \{0, 0, 0, \pm U_p(T)\} ,$$

with $U_p(T) = Nh\nu$, where $N = \dot{N}\Delta t$, with $\dot{N} = dN/dt$, is the number of photons exchanged by the system during the interval of time Δt .

For the observer in frame S_A , $Q_A^\mu = \mathcal{L}_\nu^\mu(V)Q^\nu$ and he or she assigns a linear momentum p_{pA} and a total energy E_{pA} given by :

$$\begin{aligned} p_{pA} &= -\gamma(V)\mathcal{M}_p(T)V ; \quad \mathcal{M}_p(T) = U_p(T)c^{-2} , \\ E_{pA} &= \gamma(V)U_p(T) , \end{aligned}$$

with energy function

$$U_{pA}(T) = \|Q^\mu\| = \left[E_{pA}^2 - c^2 p_{pA}^2 \right]^{1/2} = U_p(T) .$$

Doppler relativistic and aberration effects will provide a physical interpretation for the linear momentum associated with the heat in S_A (Sec. 7.2).

6 First Law of Relativistic Thermodynamics

According to the Generalised Asynchronous Formulation, there is (at least instantaneously) a frame S_0 in which the 4-vector energy function, initially U_i^μ and finally U_f^μ , work W^μ and heat Q^μ during a process may be expressed as:

$$\begin{aligned} U^\mu &= \{0, 0, 0, U\} , \\ W^\mu &= \{0, 0, 0, W\} , \\ Q^\mu &= \{0, 0, 0, Q\} . \end{aligned}$$

The First Law of classical thermodynamics leads to the *Relativistic First Law of Thermodynamics* :

Energy function variations of a system after a transition from an initial to a final equilibrium state $\Delta U^\mu = U_f^\mu - U_i^\mu$, the work performed on it during that process W^μ and the thermal radiation exchanged by the system during the process Q^μ , with every magnitude expressed as a 4-vector, is [56, 57]:

$$\Delta U^\mu = U_f^\mu - U_i^\mu = W^\mu + Q^\mu . \quad (7)$$

An observer in frame S_A has:

$$\Delta U_A^\mu = U_{fA}^\mu - U_{iA}^\mu = W_A^\mu + Q_A^\mu. \quad (8)$$

Every 4-vector in S_A is obtained from the corresponding 4-vector in S_0 through a Lorentz transformation. This procedure guarantees the covariance of the equation.

6.1 Einstein's Equation

Suppose a calorimeter were constructed so that 1 mol of deuterium nuclei could be converted into α particles and radiation – as happens in the Sun – without any observable change in the neighbourhood. No particles, radiation, or even noise would escape [58]. The binding energy associated with strong interaction is transformed into photons and kinetic energy whereas the inertia of the system remains constant during the process. For an observer in S_0 , in which the calorimeter is at rest, linear momentum is conserved. For an observer in S_A there is no change in calorimeter with velocity V . For the observer in S_A linear momentum is conserved and inertia is conserved too [23].

For an isolated system, with $W^\mu = 0$ and $Q^\mu = 0$, the energy function is conserved, and the same happens with inertia, with $\Delta U^\mu = 0$. This is the Einstein's Equation, expressed as $\Delta \mathcal{M} = \Delta U c^{-2}$ for an extended system [59].

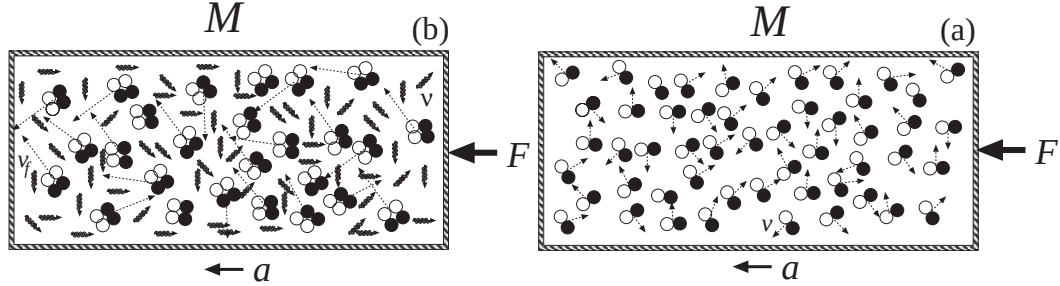


Figure 6: (a) 1 mol of ^2D nuclei, velocity v inside an isolated container, inertia $\mathcal{M}$. Under the force F the system acquires acceleration a . (b) Half mol of ^4He nuclei, with velocity $v_f > v$ and high energy photons with frequency ν . The inertia of the final system remains $\mathcal{M}$ and, under the same force F , the system acquires the same acceleration a .

In Fig. 6 the transformation of deuterium (^2D) nuclei (Sec. 3.2) into α particles (Sec. 3.1) and thermal radiation photons is shown. For the initial system $U_i = 2N_A u_D c^2 + 2N_A k_D$, with the kinetic energy of a deuterium given by $k_D = [\gamma(v) - 1]u_D$, and $\mathcal{M}_i = U_i c^{-2}$. For the whole final system $U_f = N_A u_H c^2 + N_A k_H + 2N_A h\nu$, where $k_H = [\gamma(v_f) - 1]u_H$ is the kinetic energy of a He nucleus, and $\mathcal{M}_f = U_f c^{-2}$. Einstein's Equation for the system may

be expressed as $\Delta U = 0$ or $\mathcal{M}_f = \mathcal{M}_i$. When photons leave the system, its energy function decreases as well as its inertia.

Lorentz covariance of the First Law of Relativistic Thermodynamics is based on the Principle of the Inertia of Energy or Einstein's Equation.

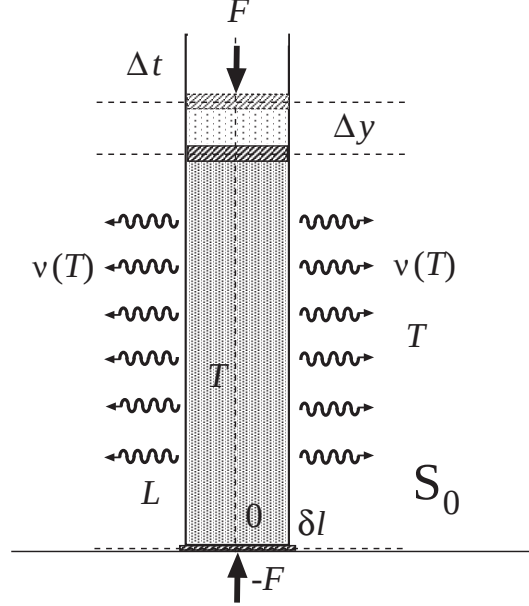


Figure 7: Vertical cylinder filled with a D_2 gas, closed by a massless piston and a fixed bottom. Isothermal compression as described by an observer in frame S_0 , in which the cylinder remains at rest as well as the thermal surroundings. The net emitted photon number is the difference between the total emitted and absorbed photons through the diathermal walls of the cylinder. Force $F_T = -F$ has displacement $-\Delta y$ and force $F_B = F$ has displacement δl . The forces act over a time interval Δt .

7 Isothermal compression of a gas

A vertical cylinder, with diathermal walls, section A and height L , contains 1 mol, N_A molecules, of a D_2 gas, enclosed by a piston (Fig. 7). The cylinder end opposed to the piston and the piston itself are supposed to be adiabatic. We assume that it is an ideal gas, the thermal equation of state being $P\mathcal{V} = N_A k_B T$, The gas is in equilibrium at pressure P_i , temperature T and volume $\mathcal{V}_i$, with $\mathcal{V}_i = RT/P_i$.

7.1 Compression in frame S_0

Frame S_0 is the frame in which the cylinder and the floor are at rest. In frame S_0 the external walls that provide the thermal radiation environment for the

cylinder, the thermal bath, are at rest too. During the compression process the centre-of-mass of the gas will be displaced, with initial and final velocity zero. Forces on the system are applied simultaneously, for a time Δt in S_0 and thermal radiation is exchanged with zero impulse in S_0 .

1. *Energy function.* As previously obtained (Sec. 3.2), for the whole gas in S_0 ($\mathbf{p}_i = 0$), the system's total energy E_i is, by definition, its energy function $U_i = E_i$ and the 4-vector initial energy function U_i^μ are:

$$U_i^\mu = \{0, 0, 0, U_i\} ; \quad U_i = E_i = \gamma(v) N_A u .$$

The energy function $U_i = U(T)$ depends on the temperature through the temperature dependence on velocity $v = v(T)$.

The energy function of an ideal gas in an isothermal process remains constant. Temperature, and the D_2 molecular speed, are constant. The 4-vector final energy function U_f^μ is:

$$U_f^\mu = \{0, 0, 0, U_f\} ,$$

with $U_f = U_i = U(T)$.

2. *Work.* In frame S_0 two forces, $F_T = -F$, on the piston and force F_B on the base of the cylinder, with $F_T = -F_B$, are applied to the system. In frame S_0 the forces F_T and F_B are applied simultaneously for a time Δt . The piston displaces by an amount $\Delta y_T = -\Delta y$ and cylinder base displaces by an amount $\Delta y_B = \delta l$, with $\delta l \approx 0$.

In frame S_0 the total impulse $\mathcal{I}$ on the gas is zero. The 'force-displacement product' W is:

$$W = F_T \Delta y_T + F_B \Delta y_B = F \Delta y + F \delta l .$$

The 4-vector work (impulse-work) W^μ in S_0 is:

$$W^\mu = \{0, 0, 0, F (\Delta y + \delta l)\} .$$

3. *Heat.* The gas is surrounded by a thermal bath at temperature T . In the monochromatic approximation the photons in the thermal reservoir has frequency $\nu(T)$. The gas and the thermal bath are mutually at rest. During the gas compression time interval Δt (assumed slow), N photons with frequency $\nu(T)$ are emitted, with zero total linear momentum. It is assumed that the photons are emitted through the vertical walls of the cylinder: $N/2$ photons are emitted in direction $\theta_+ = 0$ and $N/2$ photons

in direction $\theta_- = -\pi$. For this ensemble of N photons the total linear momentum $\mathbf{p}_p = (p_{xp}, 0, 0)$ is:

$$p_{xp} = \frac{N}{2} \frac{h\nu}{c} \cos \theta_+ + \frac{N}{2} \frac{h\nu}{c} \cos \theta_- = 0,$$

and the total energy (energy function) U_p is:

$$U_p = Nh\nu.$$

In frame S_0 the 4-vector Q^μ of thermal energy lost by the body is:

$$Q^\mu = \{0, 0, 0, -Nh\nu\},$$

From the First Law $U_f^\mu - U_i^\mu = W^\mu + Q^\mu$, one obtains $0 = F\Delta y + F\Delta l - Nh\nu$. Thus

$$Nh\nu = F\Delta y + F\delta l.$$

The configurational work provides the energy emitted as thermal radiation (photons). This is a classical thermodynamics result.

7.2 Compression in frame S_A

The previous compression process can also be described in frame S_A (Fig. 8).

In frame S_A , the corresponding 4-vectors can be obtained from S_0 4-vectors by Lorentz transformation.

$$\begin{aligned} U_{iA}^\mu &= \{-c\gamma(V)\mathcal{M}(T)V, 0, 0, \gamma(V)U(T)\}; \quad \mathcal{M}(T) = U(T)c^{-2} \\ U_{fA}^\mu &= \{-c\gamma(V)\mathcal{M}(T)V, 0, 0, \gamma(V)U(T)\}, \\ \Delta U_A^\mu &= \{0, 0, 0, 0\}, \\ W_A^\mu &= \{-c\gamma(V)\mathcal{M}_W V, 0, 0, \gamma(V)W\}; \quad \mathcal{M}_W = Wc^{-2}; \quad W = F\Delta y + F\delta l \\ Q_A^\mu &= \{-\gamma(V)\mathcal{M}_Q V, 0, 0, \gamma(V)Q\}; \quad \mathcal{M}_Q = Qc^{-2}; \quad Q = -Nh\nu. \end{aligned}$$

With the First Law expressed as $U_{fA}^\mu - U_{iA}^\mu = W_A^\mu + Q_A^\mu$ in frame S_A , the Lorentz covariance of equations ensures that the same result as in S_0 – that $Nh\nu = F(\Delta y + \delta l)$ – is obtained.

The observer in S_A may obtain the corresponding 4-vectors, U_{iA}^μ , U_{fA}^μ , W_A^μ and Q_A^μ by measuring the corresponding magnitudes, displacements Δx_A , times Δt_A , velocities, v_A , forces, F_A , frequencies, ν_A , etc., in his or her own frame.

1. *Energy function.* In order to ensure that in frame S_0 the total linear momentum is zero, for every j -th molecule with velocity components $\mathbf{v}_j = (v_{xj}, v_{yj}, 0) = (v_x, v_y, 0)$ and linear momentum $\mathbf{p}_j = \gamma(v)m\mathbf{v}_j$ (we assume for simplicity that the molecules move in $x - y$ plane), there is

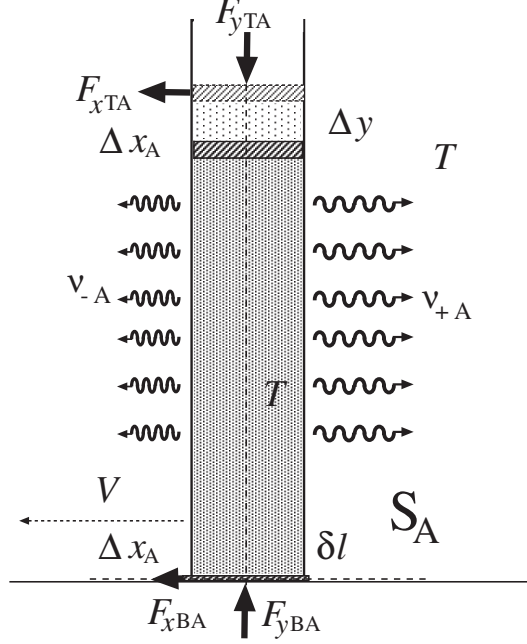


Figure 8: Vertical cylinder with a D_2 gas. Isothermal compression is given in Fig. 7 as described by an observer in frame S_A , in standard configuration to S_0 , with speed V . The forces are shown resolved into horizontal and vertical components and the photons emitted at left or right are emitted with different frequencies.

another n -th D_2 molecule with velocity components $\mathbf{v}_n = (v_{xn}, v_{yn}, 0) = (-v_x, -v_y, 0)$ and linear momentum $\mathbf{p}_n = \gamma(v)m\mathbf{v}_n$, so that $\mathbf{p}_j + \mathbf{p}_n = 0$ (Fig. 9). In frame S_0 velocities are measured simultaneously, *i. e.*, displacements $\Delta\mathbf{r}_j$ and $\Delta\mathbf{r}_n$ are measured during the same interval of time Δt . According to the Asynchronous Formulation, an observer in S_A does not measure velocities directly but instead transform to frame S_A velocities simultaneously measured in S_0 .

The velocity $\mathbf{v}_A = (v_{xA}, v_{yA}, 0)$ is obtained from the velocity $\mathbf{v}$ in S_0 from the relativistic transformations

$$v_{xA} = \frac{v_x - V}{1 - v_x V/c^2}, \quad (9)$$

$$v_{yA} = \frac{\gamma^{-1}(V) v_y}{1 - v_x V/c^2}, \quad (10)$$

together with the useful relations [7]:

$$\gamma(v_A) = \gamma(v)\gamma(V) \left[1 - \frac{v_x V}{c^2} \right], \quad (11)$$

$$\gamma(v_A) v_{xA} = \gamma(v)\gamma(V) (v_x - V), \quad (12)$$

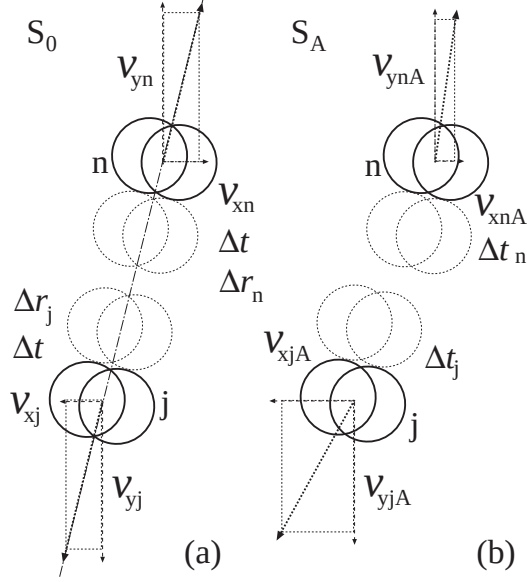


Figure 9: (a) D₂ molecules j and n with opposite velocities in frame S_0 . (b) Relativistic transformation of velocity components for molecules j and n in frame S_A .

$$\gamma(v_A) v_{yA} = \gamma(v) v_y. \quad (13)$$

It is easy to obtain total linear momentum and total energy in S_A for a pair of opposite j - n D₂ molecules, and given that there are $N_A/2$ pairs j - n , the total linear momentum p_{iA} and the total energy E_{iA} for the system in frame S_A are:

$$\begin{aligned} p_{ixA} &= -\gamma(V) \left[\frac{N_A \gamma(v) u}{c^2} \right] V, \\ p_{iyA} &= 0, \\ E_{iA} &= \gamma(V) [N_A \gamma(v) u]. \end{aligned}$$

Since $U_{iA} = \{cp_{iA}, 0, 0, E_{iA}\}$ it follows that

$$U_{iA} = \{-c\gamma(V)\mathcal{M}V, 0, 0, \gamma(V)U(T)\}$$

where $U(T) = ||U_{iA}||$ and

$$\mathcal{M}(T) = N_A \frac{u}{c^2} \gamma(v) = U(T) c^{-2},$$

is the inertia associated with the whole gas according to the observer in S_A . This result is consistent with the Principle of the Inertia of Energy [60]. The 4-vector U_{iA}^μ was previously obtained through a Lorentz transformation. A similar description can be formulated for the 4-vector final energy function U_{fA}^μ .

2. *Work.* In order to obtain the 4-vector W_A^μ in S_A relativistic transformations of time intervals, spatial displacements and forces must be used. In S_A the forces on the top and bottom of the cylinder are applied during the following space-time intervals [61]:

$$\begin{aligned}\Delta x_{TA}^\mu &= \{-\gamma(V)V\Delta t, -\Delta y, 0, c\gamma(V)\Delta t\}, \\ \Delta x_{BA}^\mu &= \{-\gamma(V)V\Delta t, \delta l, 0, c\gamma(V)\Delta t\}.\end{aligned}$$

In S_A forces are applied neither simultaneously nor during the same interval of time [62]. With force components $\mathbf{F} = (F_x, F_y, 0)$ in the frame S_0 the transformation of the force components $\mathbf{F}_A = (F_{xA}, F_{yA}, 0)$ in standard configuration is given by [63]:

$$F_{xA} = \frac{F_x - (V/c^2)(F_x v_x + F_y v_y)}{1 - v_x V/c^2}, \quad (14)$$

$$F_{yA} = \frac{\gamma^{-1}(V) F_y}{1 - v_x V/c^2}, \quad (15)$$

where $\mathbf{v} = (v_x, v_y, 0)$ is the 3-vector velocity of the body in S_0 . For vertical forces ($F_x = 0$) in S_0 ,

$$\begin{aligned}F_{xTA} &= -\left(\frac{V}{c^2}\right)(F_T v_{yT}) = \left(\frac{V}{c^2}\right)(F v_{yT}) \quad [\text{as } F_T = -F] \\ F_{yTA} &= -\gamma^{-1}(V)F.\end{aligned}$$

where $v_{yT} = -\Delta y/\Delta t$. The observer in S_A obtains a horizontal component of force as well as a vertical component. For a force applied to the cylinder base:

$$\begin{aligned}F_{xBA} &= -\left(\frac{V}{c^2}\right)(F_B v_{yd}) = -\left(\frac{V}{c^2}\right)(F v_{yd}) \quad [\text{as } F_B = F] \\ F_{yBA} &= \gamma^{-1}(V)F,\end{aligned}$$

where $v_{yd} = \delta l/\Delta t \approx 0$.

The total impulse $\mathbf{I}_A = (I_{xA}, I_{yA}, 0)$ and ‘force-displacement product’ (work) W_A in frame S_A are given by:

$$\begin{aligned}I_{xA} &= F_{xTA}\Delta t_{TA} + F_{xBA}\Delta t_{BA} = -\gamma(V)V\frac{F\Delta y + F\delta l}{c^2}, \\ I_{yA} &= F_{yTA}\Delta t_{TA} + F_{yBA}\Delta t_{BA} = 0, \\ W_A &= F_{xTA}\Delta x_{TA} + F_{xBA}\Delta x_{BA} + F_{yTA}\Delta y_{TA} + F_{yBA}\Delta y_{BA} \\ &= \gamma(V)(F\Delta y + F\delta l),\end{aligned}$$

with

$$W_A^\mu = \{cI_{xA}, cI_{yA}, 0, W_A\}. \quad (16)$$

This is the same result for W_A^μ as previously obtained by using a Lorentz transformation on W^μ .

The observer in S_A obtains a non-zero impulse on the body. According to the Principle of Inertia of Energy, the observer in S_A associates inertia, given by $(F\Delta y + F\delta l) c^{-2}$, to the work $W = F\Delta y + F\delta l$ performed in S_0 .

3. *Heat.* In order to ensure zero total linear momentum for the emitted photons in frame S_0 , for each j -th photon emitted with angle $\theta_j = 0$ there must exit another n -th photon emitted with angle $\theta_n = -\pi$, both photons having frequency ν ,

$$\begin{aligned} Q_j^\mu &= \{h\nu \cos \theta_j, h\nu \sin \theta_j, 0, h\nu\} , \\ Q_n^\mu &= \{h\nu \cos \theta_n, h\nu \sin \theta_n, 0, h\nu\} . \end{aligned}$$

According to the relativistic Doppler effect, the frequency ν_A of a photon emitted in S_0 with frequency ν and direction $(\cos \theta, \sin \theta, 0)$ is given by:

$$\nu_A = \gamma(V) [1 - \beta(V) \cos \theta] \nu . \quad (17)$$

The relativistic Aberration effect [64] indicates that the direction $(\cos \theta_A, \sin \theta_A, 0)$ as measured in S_A for the same photon is given by:

$$\cos \theta_A = \frac{\cos \theta - \beta(V)}{1 - \beta(V) \cos \theta} , \quad (18)$$

$$\sin \theta_A = \frac{\gamma^{-1}(V) \sin \theta}{1 - \beta(V) \cos \theta} . \quad (19)$$

In frame S_A this pair j - n of oppositely emitted photons (in S_0) makes the following contributions to the linear momentum and energy as:

$$\begin{aligned} p_{x(j+n)_{pA}} &= -\gamma(V) \beta(V) \left[\frac{2h\nu}{c} \right] = -\gamma(V) \left[\frac{2h\nu}{c^2} \right] V , \\ p_{y(j+n)_{pA}} &= 0 , \\ E_{(j+n)_{pA}} &= \gamma(V) [2h\nu] . \end{aligned}$$

With $N/2$ pairs of oppositely emitted photons, the total linear momentum and energy in S_A are given by:

$$\begin{aligned} p_{x_{pA}} &= -\gamma(V) \left[\frac{Nh\nu}{c^2} \right] V , \\ p_{y_{pA}} &= 0 , \\ E_{pA} &= \gamma(V) Nh\nu . \end{aligned}$$

The 4-vector $Q_A^\mu = \{-cp_{x_{pA}}, -cp_{y_{pA}}, 0, -E_{pA}\}$ is then given by:

$$Q_A^\mu = \left\{ c\gamma(V) \left[\frac{Nh\nu}{c^2} \right] V, 0, 0, -\gamma(V) [Nh\nu] \right\} .$$

This is the same result as previously obtained by using a Lorentz transformation on 4-vector Q^μ .

The total energy for photons E_{pA} as measured in S_A , and the total energy for photons U_p as measured in S_0 , are related by:

$$E_{pA} = \gamma(V)U_p. \quad (20)$$

The norm $||Q_A^\mu||$ of 4-vector Q_A^μ is an invariant, with:

$$||Q_A^\mu|| = U_p = Nh\nu. \quad (21)$$

If heat is defined as the emitted (or absorbed) energy carried by the photons in frame S_0 , in which the interchanged with a thermal medium that is at rest and so has zero linear momentum – as is defined (implicitly) in classical thermodynamics –, then Q is the norm of some 4-vector $Q = ||Q_A^\mu||$ and it is invariant. In any case, it is the 4-vector Q^μ that has physical meaning and not any of its components.

In frame S_A the total energy, E_A , linear momentum, p_A and energy function U remain constant during the compression process of the ideal gas enclosed in the cylinder –as they remain constant in frame S_0 . On the other hand, in frame S_A the set of applied forces produces a net impulse –in contrast with the net zero impulse in S_0 –, and the ensemble of emitted photons carries linear momentum –in contrast with the zero net momentum of the photons in S_0 –. In this case, the impulse and work provided by the applied external forces on the gas are transmitted to the ensemble of emitted photons, with the net impulse and work denoted by the 4-vector W_A^μ transformed in the linear momentum and energy of photons represented by 4-vector Q_A^μ . This transformation may be viewed as a relativistic generalization of the complete transformation of work into heat in an isothermal process on an ideal gas. The description of the process in frame S_0 is the usual description in classical thermodynamics, with energy associated with the heat but without linear momentum.

8 Conclusions

1. In classical physics, Newton's Second Law and the First Law of Thermodynamics are related, in Galilean covariant form, (with constant mass m) by:

$$\begin{aligned} \sum_k F_{xk} dt &= m dv_x, \\ \sum_k F_{yk} dt &= m dv_y, \end{aligned} \quad (22)$$

$$\begin{aligned} \sum_k F_{zk} dt &= m dv_z, \\ \delta W_{\text{ext}} + \delta Q &= dK_{\text{cm}} + dU. \end{aligned} \quad (23)$$

In the Minkowski description of the Special Theory of Relativity, Eqs. (22)-(23) merge in the Relativistic Thermodynamics First Law, expressed as $\Delta U^\mu = W^\mu + Q^\mu$, (S_0 is a reference frame instantaneously at rest, with system velocities $v_i = 0$ and $v_f = v$), with:

$$\begin{aligned}\sum_k F_{xk} dt &= d[\gamma(v)\mathcal{M}(T)v_x] , \\ \sum_k F_{yk} dt &= d[\gamma(v)\mathcal{M}(T)v_u] , \\ \sum_k F_{zk} dt &= d[\gamma(v)\mathcal{M}(T)v_z] , \\ \left(\sum_k \mathbf{F}_k \cdot \mathbf{v}_k\right) dt + \dot{N} h\nu dt &= d[\gamma(v)U(T)] ,\end{aligned}\quad (24)$$

where $\dot{N} = dN/dt$ is the rate of exchange of photons between the body and the thermal reservoir. This is a Lorentz covariant equation, with the Einstein's Equation $\mathcal{M}(T) = U(T)c^{-2}$, playing a central role.

- Let the 4-vector velocity be $V^\mu = \gamma(V)\{V_x, V_y, V_z, c\}$. The 4-vector $p_A^\mu = c^{-1}U_A^\mu = \{\gamma(V)\mathcal{M}V_x, \gamma(V)\mathcal{M}V_y, \gamma(V)\mathcal{M}V_z, \gamma(V)Uc^{-1}\}$, is then:

$$p_A^\mu = \frac{U}{c^2} V^\mu . \quad (25)$$

This equation generalises the equation $\mathbf{p} = (E/c^2)\mathbf{V}$ for a particle moving with velocity $\mathbf{V}$.

- In frame S_0 the direction vector is $1^\mu = \{0, 0, 0, 1\}$. Let the contravariant 4-vector direction in S_A be n^μ , defined as

$$n^\mu = \mathcal{L}_\nu^\mu(V)1^\nu = c^{-1}\{-\gamma(V)V, 0, 0, \gamma(V)c\}$$

and let

$$n_\mu = c^{-1}\{\gamma(V)V, 0, 0, \gamma(V)c\}$$

be the covariant 4-vector direction, with $n_\mu n^\mu = 1$, where V is the speed of frame S_A with respect to frame S_0 . The 4-vector n_μ satisfy the equations

$$\begin{aligned}n_\mu W_A^\mu &= W , \\ n_\mu Q_A^\mu &= Q .\end{aligned}$$

The projection of the First Law in frame S_A onto the 4-vector direction n_μ is [49]:

$$n_\mu [U_{fA}^\mu - U_{iA}^\mu = W_A^\mu + Q_A^\mu] \rightarrow U_f - U_i = W + Q .$$

This projection reduces the Lorentz covariant First Law expressed in terms of 4-vectors to the First Law of classical thermodynamics.

4. We note that:

- (i) by requiring that the 4-vector velocity in S_A v_A^μ and the 4-vector velocity in S_0 v^μ are related by

$$\begin{aligned} v_A^\mu &= \gamma(v_A) \{v_{xA}, v_{yA}, v_{zA}, c\} , \\ v^\mu &= \gamma(v) \{v_x, v_y, v_z, c\} , \\ v_A^\mu &= \mathcal{L}_\nu^\mu(V) v^\nu , \end{aligned}$$

the relativistic transformations of the velocities between frame S_0 and S_A , Eqs. (9)-(10) and Eqs. (11)-(13) are obtained,

- (ii) by requiring that the 4-vector Minkowski force in S_A F_A^μ and the 4-vector Minkowski force in S_0 F^μ are related by

$$\begin{aligned} F_A^\mu &= \gamma(v_A) \{F_{xA}, F_{yA}, F_{zA}, (\mathbf{F}_A \cdot \mathbf{v}_A)\} , \\ F^\mu &= \gamma(v) \{F_x, F_y, F_z, (\mathbf{F} \cdot \mathbf{v})\} , \\ F_A^\mu &= \mathcal{L}_\nu^\mu(V) F^\nu \end{aligned}$$

the relativistic force transformation Eqs. (14)-(15) between frames S_0 and S_A are obtained; and

- (iii) by requiring that the 4-vector S_A Q_A^μ and the 4-vector velocity in S_0 Q^μ are related by

$$\begin{aligned} Q_A^\mu &= h\nu_A \{\cos \theta_A, \sin \theta_A, 0, 1\} , \\ Q^\mu &= h\nu \{\cos \theta, \sin \theta, 0, 1\} , \\ Q_A^\mu &= \mathcal{L}_\nu^\mu(V) Q^\nu , \end{aligned}$$

the relativistic Doppler effect, Eq. (17) and the aberration effect, Eqs. (18)-(19), between frames S_0 and S_A are obtained.

So, the demonstration that the 4-vectors U_A^μ , W_A^μ and Q_A^μ can be obtained from the relativistic transformations of the velocities, forces and Doppler or aberration effects, from the corresponding U^μ , W^μ and Q^μ shows the coherence of the formalism of Relativistic Thermodynamics based on the Principle of Inertia of Energy and the Asynchronous Formulation.

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